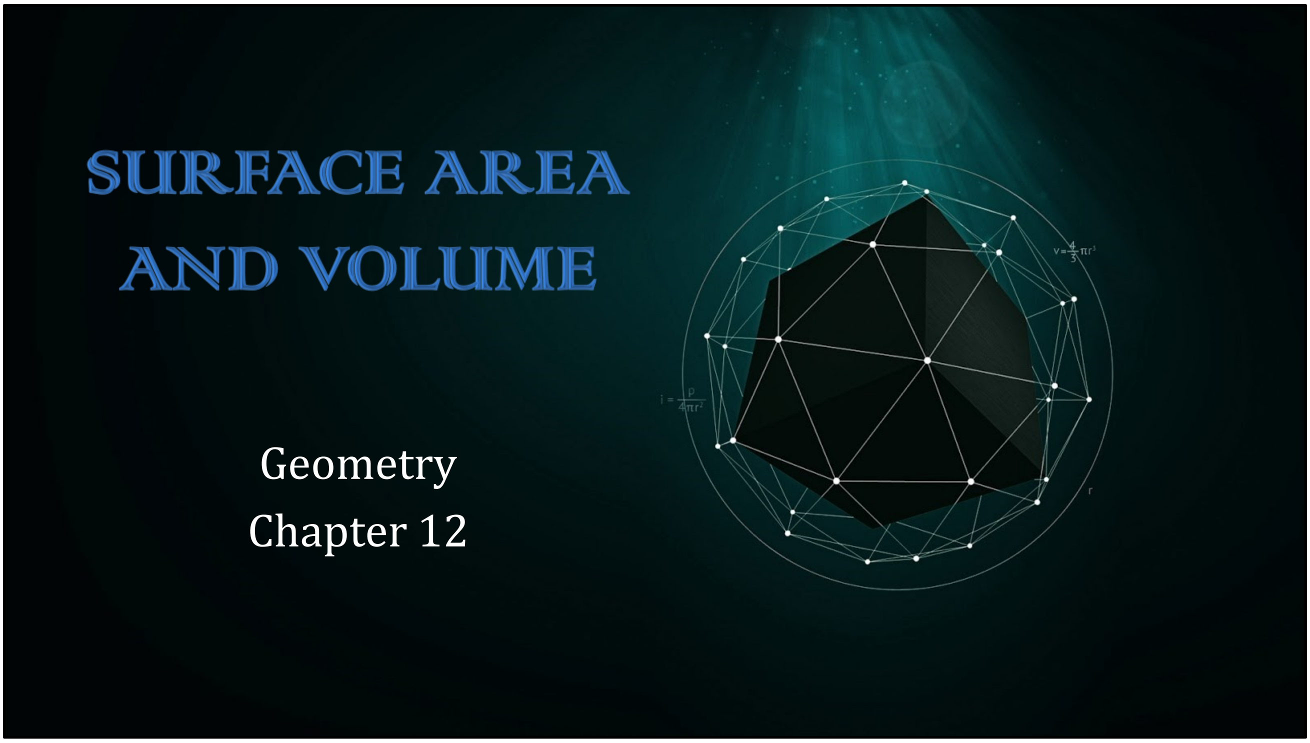


Geometry 12

- This Slideshow was developed to accompany the textbook
  - *Larson Geometry*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

Slides created by
Richard Wright, Andrews Academy
rwright@andrews.edu

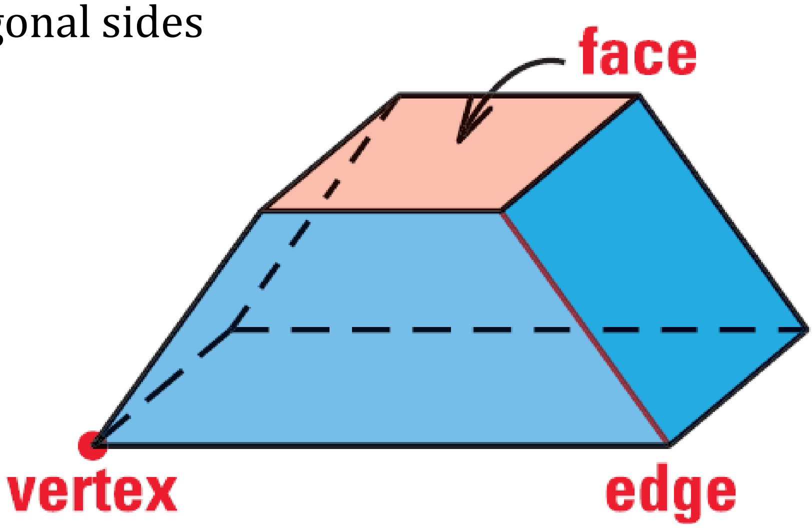
After this lesson…

- I can classify solids.
- I can use Euler’s Theorem.
- I can describe cross sections.

12.1 EXPLORE SOLIDS (12.1, NEW)

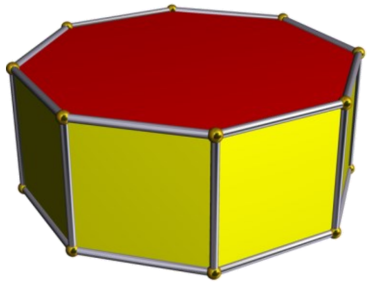
12.1 EXPLORE SOLIDS

- Polyhedron
  - ◇Solid with polygonal sides
  - ◇Flat sides
- Face
  - ◇Side
- Edge
  - ◇Line segment
- Vertex
  - ◇Corner

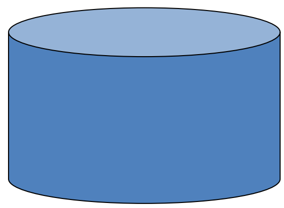


12.1 EXPLORE SOLIDS

- Prism
  - Polyhedron with two congruent surfaces on parallel planes (the 2 ends (bases) are the same)
  - Named by bases (i.e. rectangular prism, triangular prism)

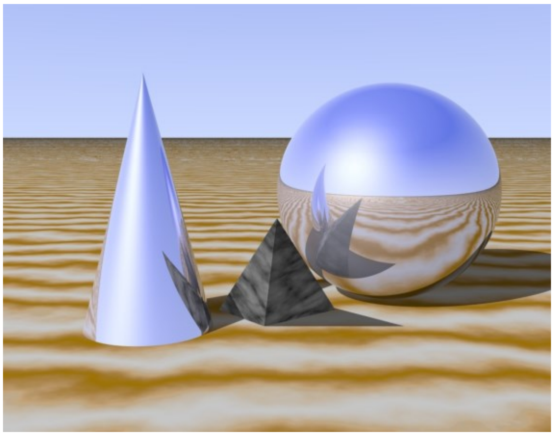


- Cylinder
  - Solid with congruent circular bases on parallel planes (not a polyhedron)



12.1 EXPLORE SOLIDS

- **Pyramid →** polyhedron with all but one face intersecting in one point
- **Cone →** circular base with the other surface meeting in a point (kind of like a pyramid)
- **Sphere →** all the points that are a given distance from the center



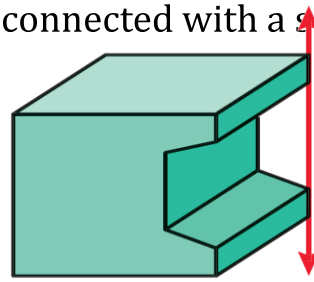
12.1 EXPLORE SOLIDS

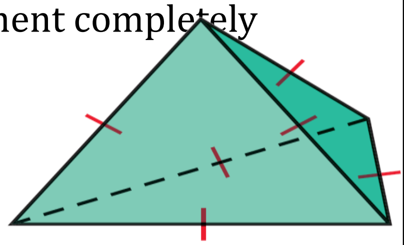
Euler's Theorem

The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by

$$F + V = E + 2$$

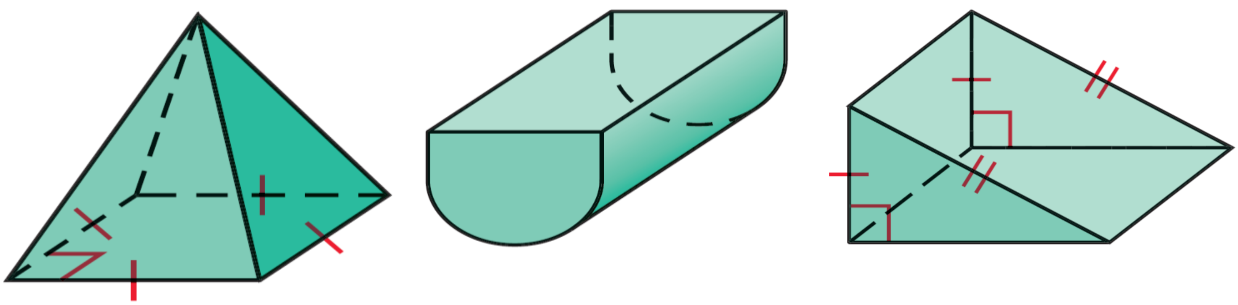
- Convex
  - Any two points can be connected with a segment completely inside the polyhedron
- Concave
  - Not convex
  - Has a "cave"





12.1 EXPLORE SOLIDS

- Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges and describe as convex or concave



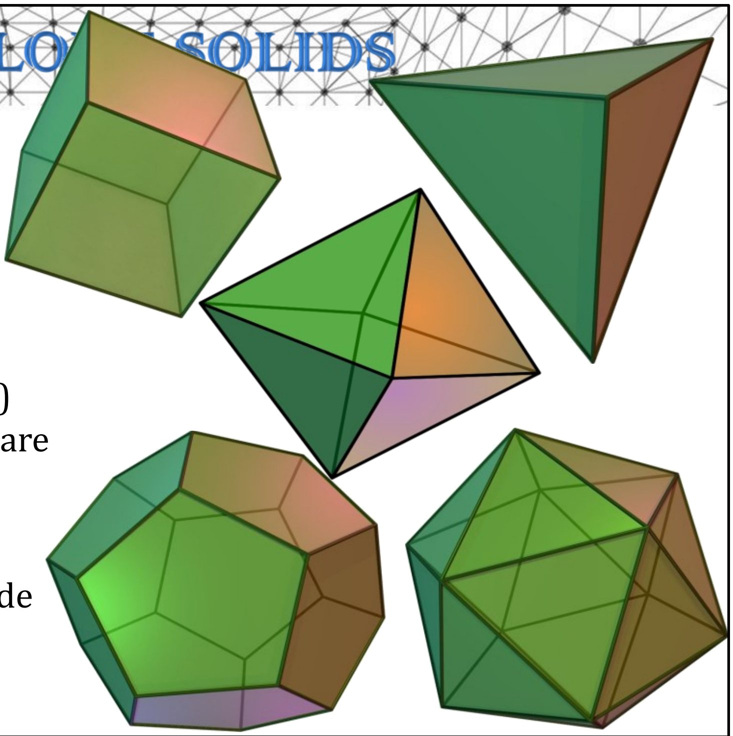
Polyhedron; Square Pyramid; 5 faces, 5 vertices, 8 edges; convex

Not a Polyhedron

Polyhedron; Triangular Prism; 5 faces, 6 vertices, 9 edges; convex

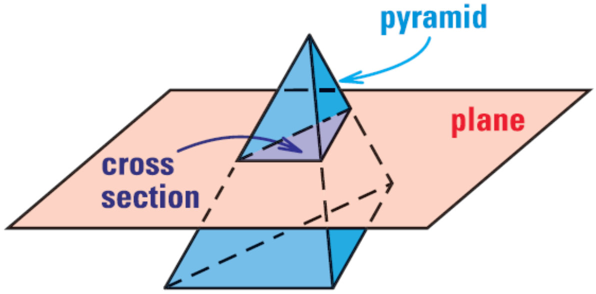
12.1 EXPLO[illegible] SOLIDS

- Regular polyhedron
  - ◇ Polyhedron with congruent regular polygonal faces
- Only 5 types (**Platonic solids**)
  - ◇ **Tetrahedron** → 4 faces (triangular pyramid)
  - ◇ **Hexahedron** → 6 faces (cube)
  - ◇ **Octahedron** → 8 faces (2 square pyramids put together)
  - ◇ **Dodecahedron** → 12 faces (made with pentagons)
  - ◇ **Icosahedron** → 20 faces (made with triangles)



12.1 EXPLORE SOLIDS

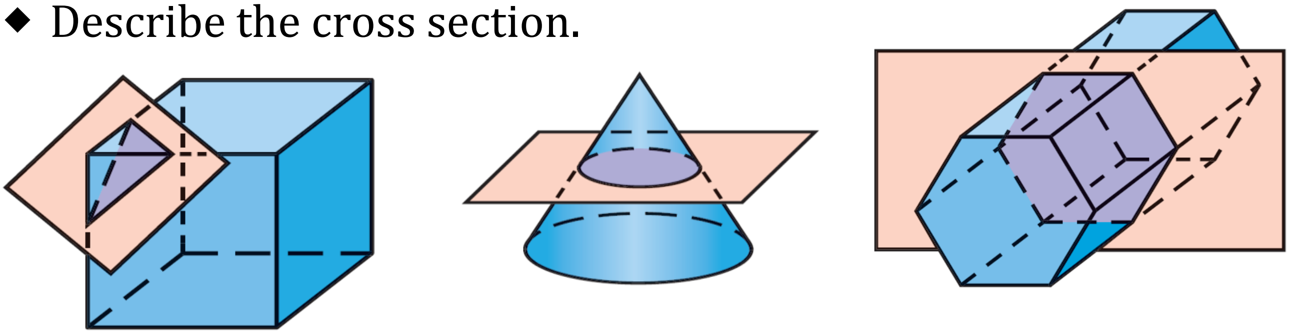
- Cross Section
  - ◇Imagine slicing a very thin slice of the solid
  - ◇The cross section is the 2-D shape of the thin slice



12.1 EXPLORE SOLIDS

- Find the number of faces, vertices, and edges of a regular dodecahedron. Check with Euler's Theorem.

- Describe the cross section.



Triangle

Circle

Hexagon

After this lesson…

- I can draw a net for a polyhedron.
- I can find the surface area of prisms and cylinders

12.2 SURFACE AREA OF PRISMS AND CYLINDERS

12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Surface area = sum of the areas of each surface of the solid
  - ◇In order to calculate surface area it is sometimes easier to draw all the surfaces

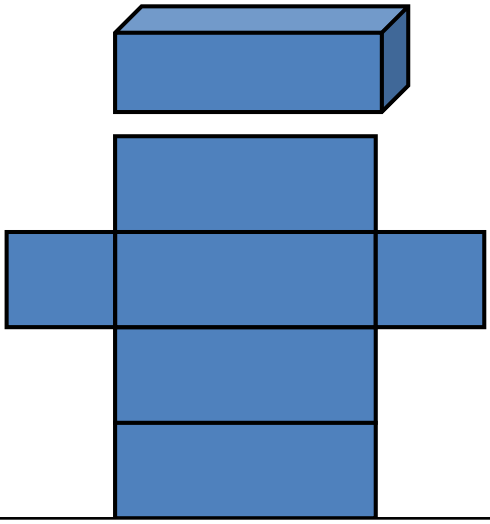
Some sports relie on having very little friction. In biking for example, the smaller the surface area of the tires, the less friction there is. And thus the faster the rider can go.

→ Draw the top triangle first (for some triangles you may have to count a horizontal space as 2)

12.2 SURFACE AREA OF PRISMS AND CYLINDERS

Nets

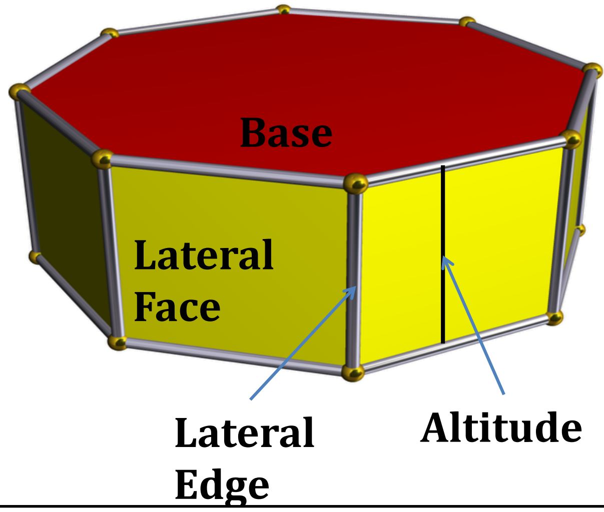
- Imagine cutting the three dimensional figure along the edges and folding it out.
- Start by drawing one surface, then visualize unfolding the solid.
- To find the surface area, add up the area of each of the surfaces of the net.



12.2 SURFACE AREA OF PRISMS AND CYLINDERS

Parts of a right prism

- **Bases →** parallel congruent surfaces (the ends)
- **Lateral faces →** the other faces (they are parallelograms)
- **Lateral edges →** intersections of the lateral faces (they are parallel)
- **Altitude →** segment perpendicular planes containing the two bases with an endpoint on each plane
- **Height →** length of the altitude



12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Right prism
  - Prism where the lateral edges are altitudes

- Oblique prism
  - Prism that isn't a right prism

12.2 SURFACE AREA OF PRISMS AND CYLINDERS

Lateral Area (L) of Prisms

- Area of the Lateral Faces

- $L = Ph$
  - ◇ L = Lateral Area
  - ◇ P = Perimeter of base
  - ◇ h = Height

You can find the surface area by adding up the areas of each surface, but if you could use a formula, it would be quicker

All the lateral surfaces are rectangles

Area = bh

Add up the areas L = ah + bh + ch + … + dh

L = (a + b + c + … + d)h

Perimeter of base = a + b + c + … + d

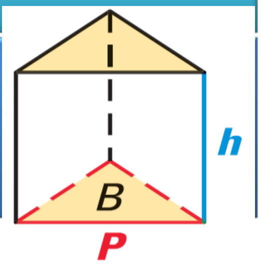
L = Ph

- Base Area (B)
  - ◇In a prism, both bases are congruent, so you only need to find the area of one base and multiply by two

Surface Area of a Right Prism

$$S = 2B + Ph$$

Where S = surface area, B = base area, P = perimeter of base, h = height of prism



12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Draw a net for a triangular prism.

- Find the lateral area and surface area of a right rectangular prism with height 7 inches, length 3 inches, and width 4 inches.

/□\
□
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$$P = 2(3) + 2(4) = 14$$
$$L = (14)(7) = 98$$
$$B = 3 \cdot 4 = 12$$
$$A = 2(12) + 14(7) = 122$$

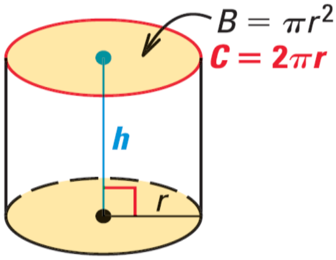
12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Cylinders are the same as prisms except the bases are circles
  - Lateral Area = $L = 2\pi rh$

Surface Area of a Right Cylinder

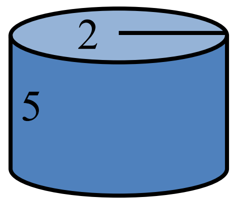
$$S = 2\pi r^2 + 2\pi rh$$

Where S = surface area, r = radius of base, h = height of prism



12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- The surface area of a right cylinder is 100 cm^2. If the height is 5 cm, find the radius of the base.
- Draw a net for the cylinder and find its surface area.



$$100 = 2\pi r^2 + 2\pi r(5)$$
$$100 = 2\pi r^2 + 10\pi r$$
$$0 = 2\pi r^2 + 10\pi r - 100$$
$$0 = r^2 + 5r - 15.915$$
$$r = \frac{-5 \pm \sqrt{5^2 - 4(1)(-15.915)}}{2(1)}$$
$$r = \frac{-5 \pm \sqrt{88.662}}{2}$$
$$r = 2.2, -7.2$$

Only 2.2 makes sense because the radius must be positive

$$S = 2\pi 2^2 + 2\pi(2)(5)$$
$$S = 8\pi + 20\pi = 28\pi$$

After this lesson…

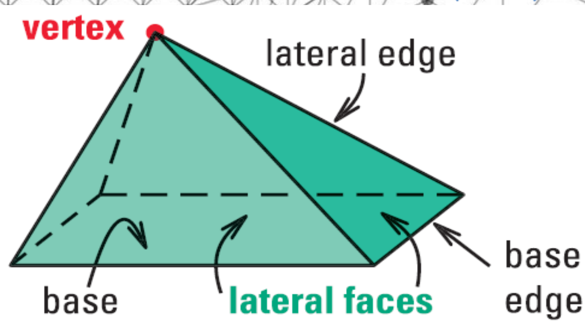
- I can find the surface area of pyramids.
- I can find the surface area of cones.

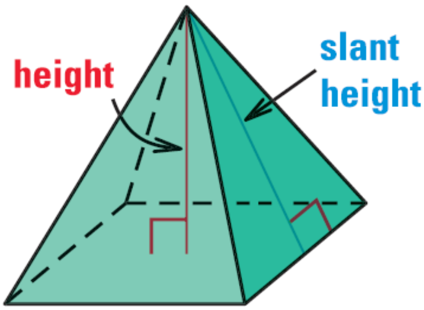
12.3 SURFACE AREA OF PYRAMIDS AND CONES (12.4)

12.3 SURFACE AREA OF PYRAMIDS AND CONES (12.4)

Pyramids

- All faces except one intersect at one point called **vertex**
- The **base** is the face that does not intersect at the vertex
- **Lateral faces** → faces that meet in the vertex
- **Lateral edges** → edges that meet in the vertex
- **Altitude** → segment that goes from the vertex and is perpendicular to the base



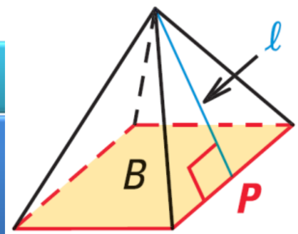


12.3 SURFACE AREA OF PYRAMIDS AND CONES (12.4)

- **Regular pyramid →** base is a regular polygon and the vertex is directly above the center of the base
  - ◇ In a regular pyramid, all the lateral faces are congruent isosceles triangles
  - ◇ The height of each lateral face is called the **slant height** (ℓ)
- **Lateral Area →** $L = \frac{1}{2}P\ell$

Surface Area of a Regular Pyramid

$$S = B + \frac{1}{2}P\ell$$

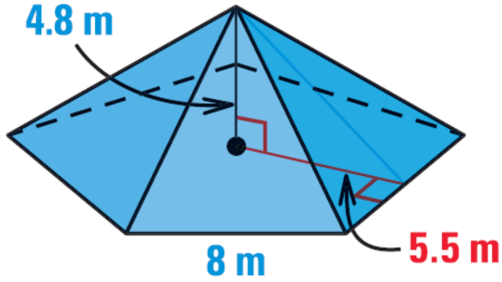


Where B = base area, P = base perimeter, ℓ = slant height

Lateral area is ½ because the sides are triangles.

12.3 SURFACE AREA OF PYRAMIDS AND CONES (12.4)

- Find the surface area of the regular pentagonal pyramid.



Base Area

$$B = \frac{1}{2}Pa$$

$$B = \frac{1}{2}(5 \cdot 8)(5.5) = 110$$

$$\ell^2 = 5.5^2 + 4.8^2$$

$$\ell = 7.3$$

$$S = B + \frac{1}{2}P\ell$$

$$S = 110 + \frac{1}{2}(5 \cdot 8)(7.3) = 256$$

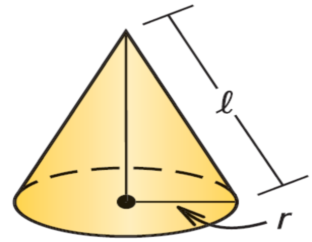
Cones

- Cones are just like pyramids except the base is a circle
- Lateral Area = $\pi r \ell$

Surface Area of a Right Cone

$$S = \pi r^2 + \pi r \ell$$

Where r = base radius, ℓ = slant height



- The So-Good Ice Cream Company makes Cluster Cones. For packaging, they must cover each cone with paper. If the diameter of the top of each cone is 6 cm and its slant height is 15 cm, what is the area of the paper necessary to cover one cone?

Looking for lateral area.

$$L = \pi 3(15) = 141.4\ cm^2$$

After this lesson…

- I can find volumes of prisms and cylinders.
- I can solve real-life problems involving volumes of prisms and cylinders.

12.4 VOLUME OF PRISMS AND CYLINDERS (12.2)

12.4 VOLUME OF PRISMS AND CYLINDERS (12.2)

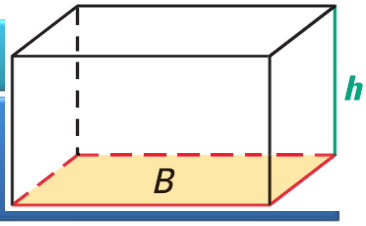
- Create a right prism using geometry cubes
- Count the lengths of the sides
- Count the number of cubes.
- Remember this to verify the formulas we are learning today.

12.4 VOLUME OF PRISMS AND CYLINDERS (12.2)

Volume of a Prism

$$V = Bh$$

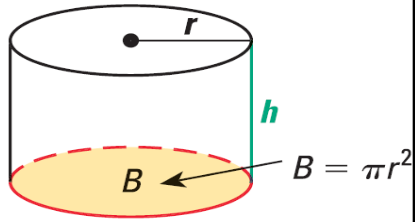
Where B = base area, h = height of prism



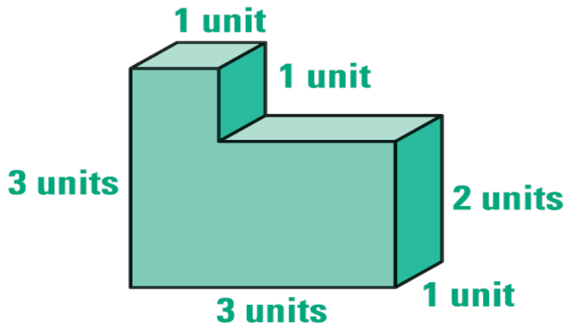
Volume of a Cylinder

$$V = \pi r^2 h$$

Where r = radius, h = height of cylinder



- Find the volume of the figure



Cut into two prisms

Top

$$V = 1(1)(1) = 1$$

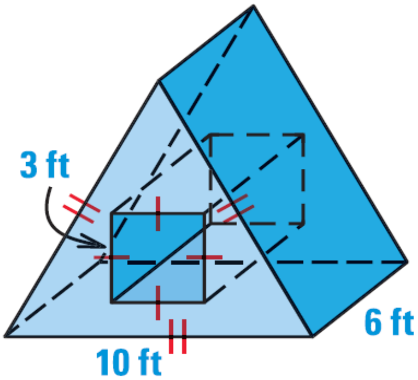
Bottom

$$V = 3(1)(2) = 6$$

Total

$$V = 1 + 6 = 7$$

12.4 VOLUME OF PRISMS AND CYLINDERS (12.2)



- Find the volume.

Base Area (front)

Find height of triangle

$$5^2 + x^2 = 10^2$$
$$25 + x^2 = 100$$
$$x^2 = 75$$
$$x = 5\sqrt{3}$$

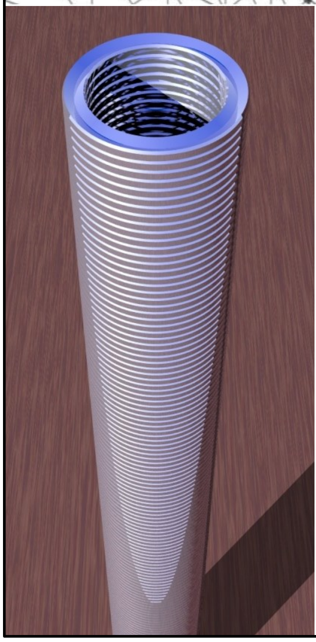
Area=triangle - square

$$B = \frac{1}{2}(10)\left(5\sqrt{3}\right) - 3^2$$
$$B = 25\sqrt{3} - 9 \approx 34.301$$

Volume = Bh

$$V = \left(25\sqrt{3} - 9\right)(6) = 150\sqrt{s} - 54 \approx 205.8$$

12.4 VOLUME OF PRISMS AND CYLINDERS (12.2)



- There are 150 1-inch washers in a box. When the washers are stacked, they measure 9 inches in height. If the inside hole of each washer has a diameter of ¾ inch, find the volume of metal in one washer.

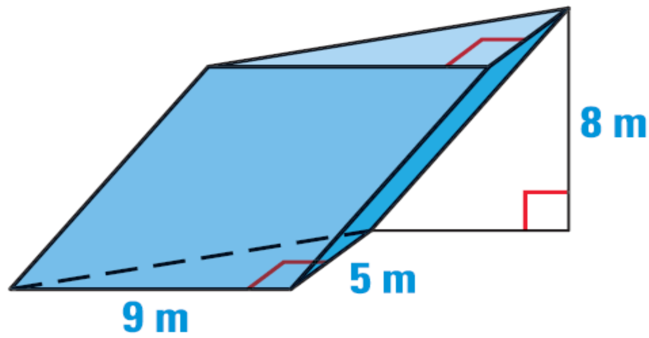
Find volume of washers without holes: $V = \pi\, ½^2\, 9 = 7.06858$
Find volume of hole: $V = \pi(3/8)^2\, 9 = 3.97608$
Find volume of washers with holes: 7.06858 – 3.97608 = 3.09251
Find volume of one washer: 3.09251/150 = 0.02 in^3

12.4 VOLUME OF PRISMS AND CYLINDERS (12.2)

Cavalieri's Principle

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

- Find the volume.



$$B = \frac{1}{2}(9)(5) = 22.5\ m^2$$
$$V = (22.5\ m^2)(8\ m) = 180\ m^3$$

After this lesson…

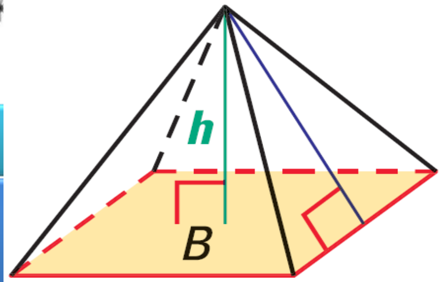
- I can find volumes of pyramids and cones.
- I can find volumes of composite solids.

12.5 VOLUME OF PYRAMIDS AND CONES (12.3, 12.4)

12.5 VOLUME OF PYRAMIDS AND CONES (12.3, 12.4)

- How much ice cream will fill an ice cream cone?
- How could you find out without filling it with ice cream?
- What will you measure?

12.5 VOLUME OF PYRAMIDS AND CONES (12.3, 12.4)



Volume of a Pyramid

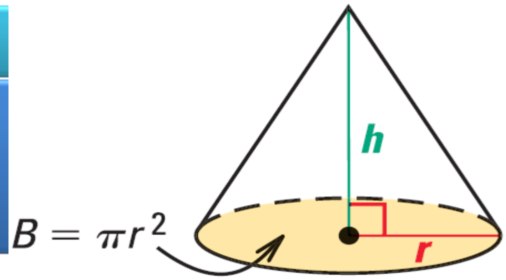
$$V = \frac{1}{3}Bh$$

Where B = base area, h = height of pyramid

Volume of a Cone

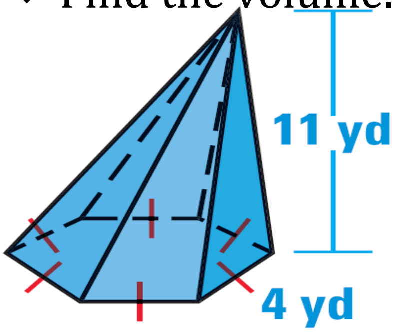
$$V = \frac{1}{3}\pi r^2 h$$

Where r = radius, h = height of cone



12.5 VOLUME OF PYRAMIDS AND CONES (12.3, 12.4)

- Find the volume.



$$B = \frac{1}{2}Pa$$

$$\frac{1}{2}central\ angle = \frac{1}{2}\left(\frac{360}{6}\right) = 30$$

$$\tan 30 = \frac{2}{a}$$

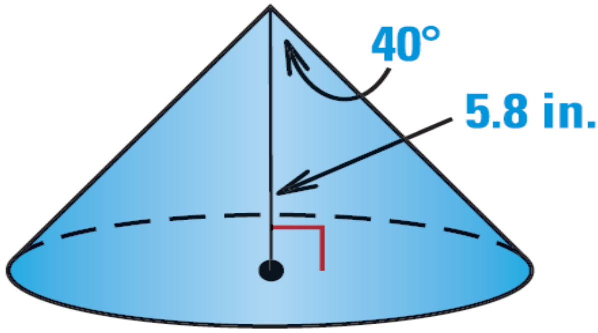
$$a = \frac{2}{\tan 30} = 3.464$$

$$B = \frac{1}{2}(4 \cdot 6)(3.464) = 41.569$$

$$V = \frac{1}{3}(41.569)(11) = 152.42$$

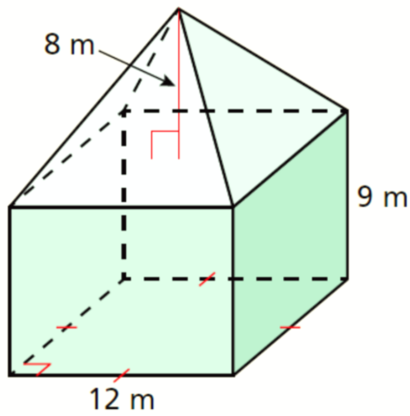
12.5 VOLUME OF PYRAMIDS AND CONES (12.3, 12.4)

- Find the volume.



$$\tan 40 = \frac{r}{5.8}$$
$$r = 5.8 \cdot \tan 40 = 4.8668$$
$$V = \frac{1}{3}\pi 4.8668^2 \cdot 5.8 = 143.86$$

- Find the volume of the composite solid.



Pyramid:

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3}(12^2)(8) = 384\ m^3$$

Prism:

$$V = Bh$$

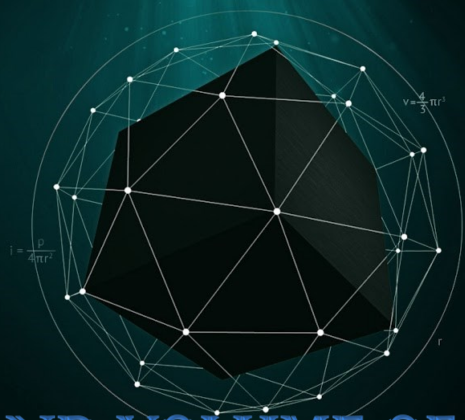
$$V = (12^2)(9) = 1296\ m^3$$

Total:

$$384\ m^3 + 1296\ m^3 = 1680\ m^3$$

After this lesson…

- I can find the surface area of spheres.
- I can find the volume of spheres.
- I can find the volumes of composite solids.

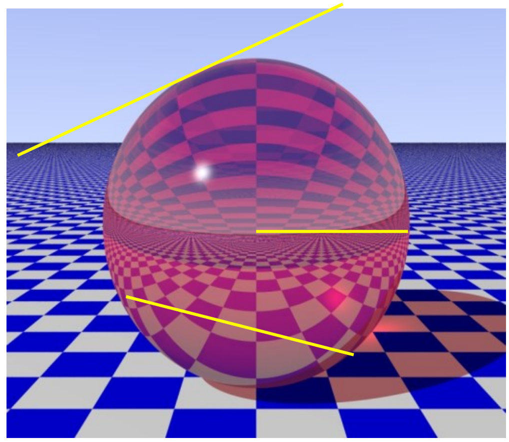


12.6 SURFACE AREA AND VOLUME OF SPHERES (12.5)

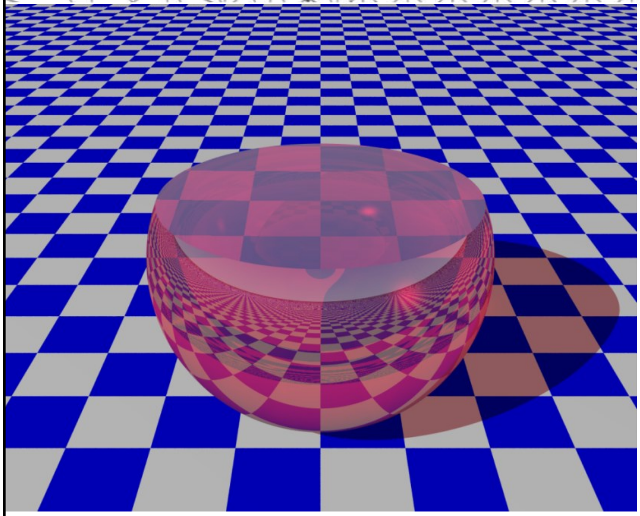
12.6 SURFACE AREA AND VOLUME OF SPHERES (12.5)

Terms

- **Sphere** → all points equidistant from center
- **Radius** → segment from center to surface
- **Chord** → segment that connects two points on the sphere
- **Diameter** → chord contains the center of the sphere
- **Tangent** → line that intersects the sphere in exactly one place

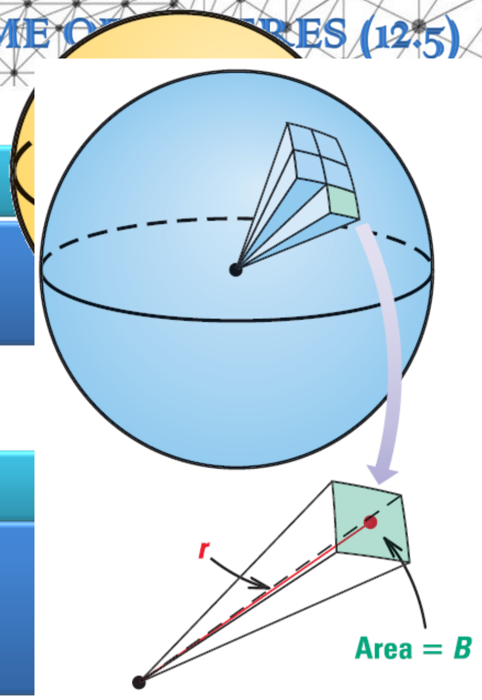


12.6 SURFACE AREA AND VOLUME OF SPHERES (12.5)



- Intersections of plane and sphere
  - ◇ **Point** → plane tangent to sphere
  - ◇ **Circle** → plane not tangent to sphere
  - ◇ **Great Circle** → plane goes through center of sphere (like equator)
    - Shortest distance between two points on sphere
    - Cuts sphere into two hemispheres

12.6 SURFACE AREA AND VOLUME O RES (12.5)



Surface Area of a Sphere

$$S = 4\pi r^2$$

Where r = radius

- If you cut 4 circles into 8ths you can put them together to make a sphere

Volume of a Sphere

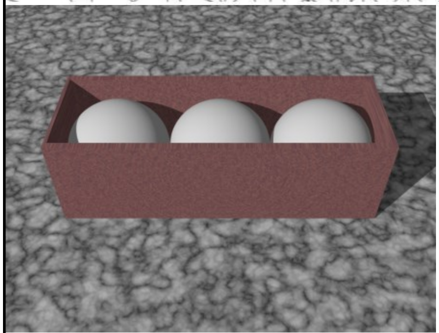
$$V = \frac{4}{3}\pi r^3$$

Where r = radius

You can think about cutting a sphere into many small regular square pyramids.

V = 1/3 Bh → the area of all the bases is $4\pi r^2$ and h = r

12.6 SURFACE AREA AND VOLUME OF SPHERES (12.5)



- Find the volume of the empty space in a box containing three golf balls. The diameter of each is about 1.5 inches. The box is 4.5 inches by 1.5 inches by 1.5 inches.

Volume of box: $4.5(1.5)(1.5) = 10.125$

Volume of each ball: $\frac{4}{3}\pi.75^3 = 1.767$

Volume of empty space: $10.125 - 3(1.767) = 4.824$

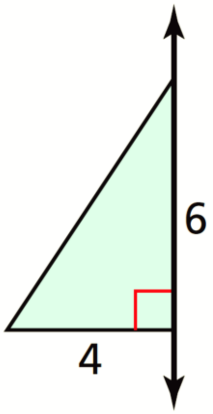
After this lesson…

- I can sketch and describe solids of revolution.
- I can find surface area and volumes of solids of revolution.
- I can form solids of revolution in the coordinate plane.

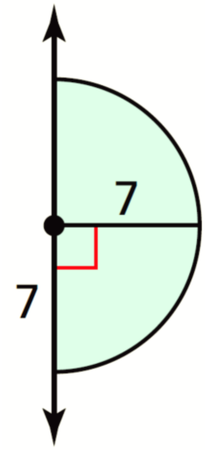
12.7 SOLIDS OF REVOLUTION (12.7)

12.7 SOLIDS OF REVOLUTION (12.7)

- Solid of Revolution
  - ◇ 3D figure form by rotating a 2D shape around an axis
  - ◇ The axis is the axis of revolution.



- Sketch the solid produced by rotating the figure around the axis.

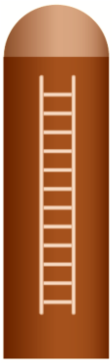


cone with a height of 6 units and a radius of 4 units

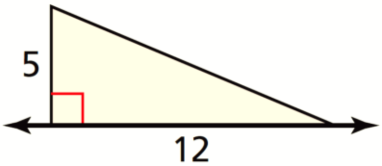
sphere with a radius of 7 units

12.7 SOLIDS OF REVOLUTION (12.7)

- Sketch a two-dimensional shape and an axis of revolution that can form the grain silo shown.



- Sketch and describe the solid produced by rotating the figure around the given axis. Then find its surface area.



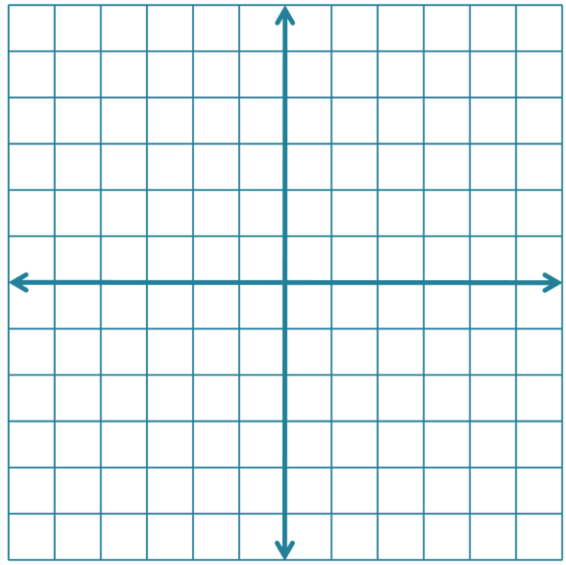
Looks like left or right half of the silo.

Makes a cone with radius 5 and height of 12
Slant height:

$$5^2 + 12^2 = \ell^2$$
$$169 = \ell^2$$
$$13 = \ell$$
$$SA = \pi r^2 + \pi r \ell$$
$$SA = \pi(5)^2 + \pi(5)(13)$$
$$SA = 90\pi \approx 282.74$$

12.7 SOLIDS OF REVOLUTION (12.7)

- Sketch and describe the solid that is produced when the region enclosed by $x = 0$, $y = 0$, and $y = x + 2$ is rotated around the x-axis. Then find the volume of the solid.



right cone with a radius of 2 units and a height of 2 units; Volume:

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(2)^2(2) = \frac{8}{3}\pi \approx 8.38$$